APPLICATION OF NONLINEAR EQUATIONS TO ECONOMIC MODELS

Author: Tracy Coyle
Originally written in 1988; reformatted and republished for reference.
University of Illinois at Chicago

Abstract

Forecasting models have been developed to predict future economic activity. In order to construct these models, several assumptions are typically made:

1. Rationality
2. Linear relationships between variables
3. Accurate estimation of initial conditions

These assumptions allow economic models to generate predictable and stable results. However, economic systems themselves are rarely stable and are fundamentally nonlinear.

This paper explores the possible application of nonlinear dynamic theory—specifically chaotic dynamics—to economic modeling. The purpose is not to develop a complete mathematical treatment of chaotic systems, but rather to examine how nonlinear dynamics may describe economic behavior more accurately than linear models.

A nonlinear relational model is introduced that illustrates interactions between consumption, investment, and interest rates. Periodic perturbations in interest rates introduce nonlinear behavior while still producing bounded and interpretable outcomes.

I. Introduction

Current economic models can often forecast economic values with reasonable short-term accuracy. However, because these models cannot fully adapt to changing economic conditions, they must be recalibrated regularly.

Although models are often constructed similarly, the data used to refine them varies according to the model builder. As a result, two models built using the same theoretical structure may produce significantly different forecasts.

Descriptions of nonlinear behavior have only recently emerged in mathematics and related sciences. These descriptions are still developing, but they suggest that many natural systems—including economic systems—may behave in fundamentally nonlinear ways.

This paper assumes that nonlinear dynamics may be applicable to economic systems and explores the implications of that assumption.

II. Current Economic Models

An examination of economic model construction reveals several similarities that limit their long-term usefulness.

A. Core Assumptions

Most economic models share three fundamental assumptions:

- Rational behavior
- Linear relationships between variables
- Accurate knowledge of initial conditions

Each of these assumptions appears reasonable in isolation. However, each also introduces limitations that can significantly affect model outcomes.

---

1. Functional Relationships

Economic models typically represent relationships using algebraic functions.

A function is defined as a mathematical operation that produces a unique output for each input.

For example:

$C = f(I)$

Where

C = consumption
I = income

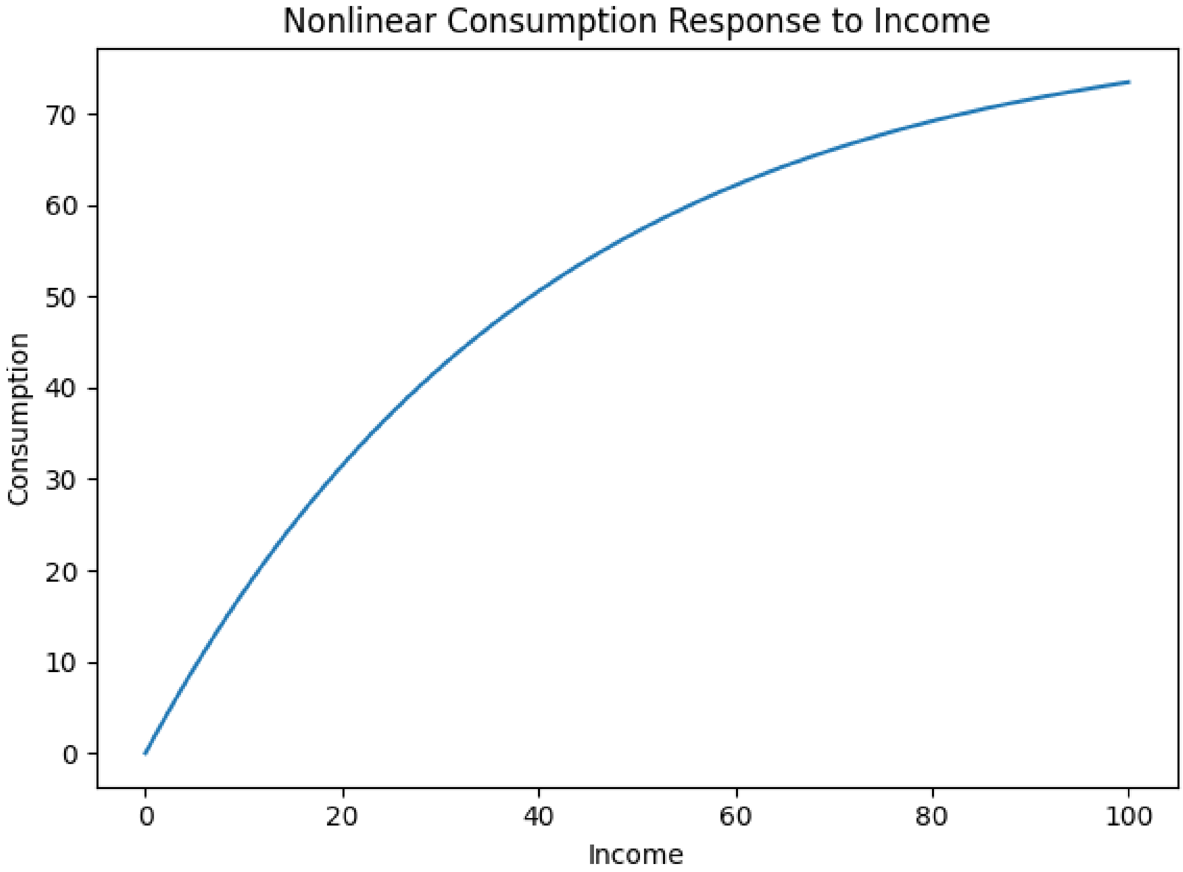
In simplified models, consumption is often expressed as a constant proportion of income.

However, this assumption becomes unrealistic at larger scales. As income increases, consumption does not increase proportionally; instead it gradually approaches a saturation point.

This produces a nonlinear relationship.

Figure 1

Nonlinear Consumption Response



Consumption increases with income but eventually levels off, illustrating that consumption behavior is nonlinear rather than proportional.

The nonlinear relationship implies that multiple income values may produce similar consumption levels. Such a relationship cannot be represented by a simple function.

Despite this, linear functions continue to be widely used because they provide single, predictable outcomes.

---

2. Rationality

Economic models typically assume individuals act as utility maximizers.

Although reasonable in theory, the assumption has limited empirical support.

Differences in individual preferences make it difficult to define a universally rational action.

For example:

Is donating to charity irrational because the money could earn interest?

The assumption of rationality is nevertheless maintained because, as Hahn noted:

Unless consistent behavior can be assumed, inquiry in the social sciences becomes extremely difficult.

---

3. Initial Conditions

Economic models also depend heavily on accurate initial conditions.

If initial values are estimated incorrectly, the resulting forecasts may diverge significantly from reality.

Nonlinear systems are particularly sensitive to initial conditions.

Small numerical errors can grow rapidly through iteration, producing outcomes unrelated to the original state.

III. Limitations of Linear Modeling

Regression analysis is the primary tool used to estimate relationships between economic variables.

Historical data is analyzed to determine how changes in one variable affect another.

However, this method assumes that historical relationships remain stable over time.

When structural changes occur, the model must be recalibrated.

As data sets grow larger, structural changes become diluted by long-term averaging. This reduces the model's ability to detect emerging trends.

As a result, many economic models describe past conditions more accurately than they predict future ones.

IV. Nonlinear Dynamics

Natural systems frequently exhibit nonlinear behavior.

Examples include:

- fluid turbulence
- weather patterns
- biological growth
- heart rhythms

These systems are governed by simple rules that produce complex behavior through iteration.

The study of such systems is known as nonlinear dynamics.

V. Chaos

Chaos refers to deterministic systems that display sensitive dependence on initial conditions.

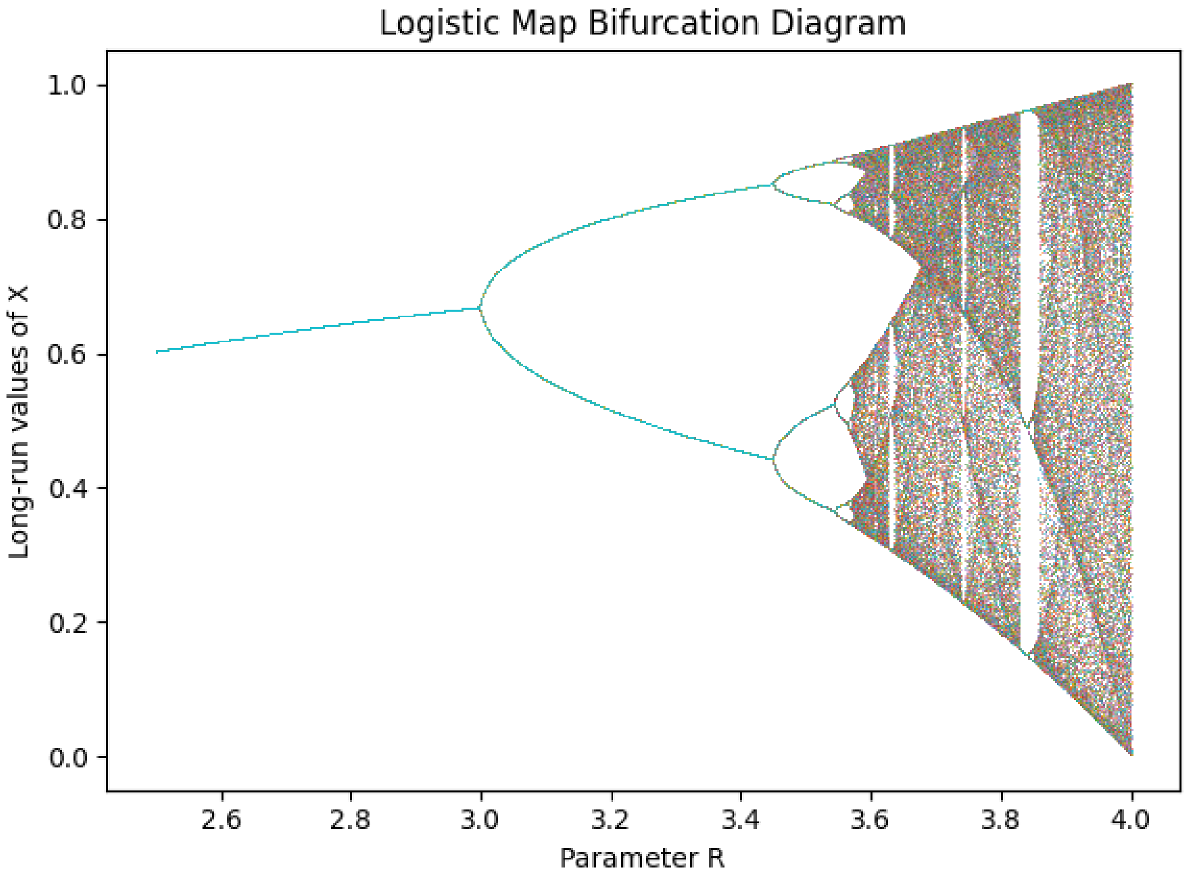
One of the most commonly studied chaotic equations is the logistic map:

$$X(t+1) = R\,X(t)\,(1 - X(t))$$

Through repeated iteration, this equation produces a range of behaviors including stability, oscillation, and chaotic fluctuations.

Figure 2

Logistic Map Bifurcation Diagram



As parameter R increases, the system transitions from stability to oscillation and eventually chaotic behavior.

Despite apparent randomness, chaotic systems often remain bounded within defined ranges.

These ranges are associated with structures known as **attractors**.

VI. Economic Chaos

To explore nonlinear economic dynamics, a simplified relational model is introduced.

Variables:

C = consumption
I = investment
R = interest rates

Relationships are represented as directional influences rather than continuous numerical values.

Example:

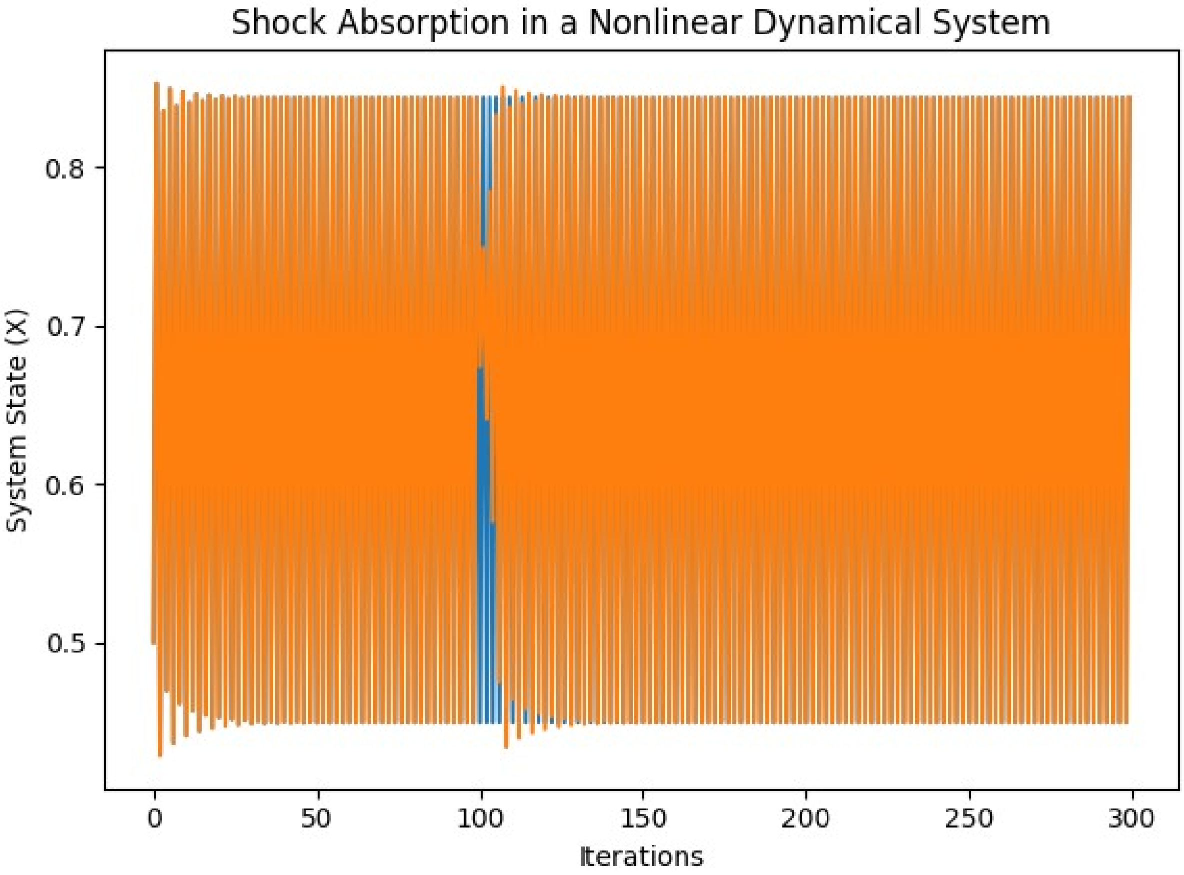
+C → +R, +I

This indicates that an increase in consumption tends to increase both interest rates and investment.

Multiple relational equations are iterated to simulate system behavior.

Figure 3

Shock Absorption in a Nonlinear System



A baseline trajectory and a perturbed trajectory diverge initially but reconverge toward the same attractor over time.

A perturbation introduced into the system temporarily alters the trajectory but does not permanently alter the system's overall behavior.

This suggests that certain economic shocks may be absorbed by the system rather than producing permanent structural change.

VII. Summary and Conclusion

Economic models rely heavily on assumptions of rationality, linear relationships, and stable initial conditions.

These assumptions simplify modeling but may fail to capture the nonlinear behavior observed in real economic systems.

Nonlinear dynamics provides an alternative framework for understanding economic behavior.

Within such systems, shocks may act as perturbations to system trajectories rather than permanent structural shifts.

The model presented here illustrates that nonlinear systems can exhibit bounded, deterministic behavior while still displaying complex dynamics.

The following reflection was written to accompany this reformatted version of the original 1988 manuscript. Aside from minor editorial adjustments for clarity and formatting, the paper itself remains substantially unchanged. The reflection provides brief context for the circumstances under which the paper was written and situates its central ideas within the subsequent development of complexity economics.

Author’s Reflection: 1988 to Present

This paper was written in 1988 as part of my undergraduate studies in economics at the University of Illinois at Chicago. At that time, chaos theory had only recently begun to enter scientific discussion outside mathematics and physics, largely through the popularization of nonlinear dynamics in works such as James Gleick’s *Chaos*. The implications of these ideas for economic systems were not widely explored within economics departments.

My interest in nonlinear systems emerged from a simple discomfort with a foundational assumption repeated throughout economic instruction: that the effects of a variable could be examined by holding all other variables constant. This assumption—useful pedagogically—seemed incompatible with the reality of economic systems, where variables interact continuously and where any change propagates through a network of relationships.

The paper therefore began as a question: if economic systems are inherently interconnected and nonlinear, why should linear models dominate economic forecasting? The goal was not to replace existing models but to explore whether nonlinear dynamic systems might provide a more accurate conceptual framework for understanding economic behavior.

The simple simulations presented in the paper suggested that perturbations introduced into a nonlinear system could be absorbed over time as the system returned toward a bounded dynamic pattern. This observation led to a tentative conclusion that certain economic shocks—particularly supply shocks—might not produce permanent structural changes in the system but instead act as temporary perturbations within a bounded dynamic process.

At the time, this conclusion ran counter to several prevailing assumptions in macroeconomic modeling, particularly the treatment of certain shocks as effectively permanent shifts in economic equilibrium. The paper did not attempt to prove that nonlinear dynamics governed economic systems, but it suggested that such systems might behave in ways fundamentally different from those predicted by linear equilibrium models.

In the decades since this paper was written, the study of economic systems as complex adaptive systems has expanded significantly. Research associated with the Santa Fe Institute, particularly the work of W. Brian Arthur and others, has developed the field now known as complexity economics. Agent-based modeling, network theory, and computational simulation have become increasingly important tools for exploring economic dynamics that cannot be easily captured by linear analytical models.

Many of the ideas that were speculative in 1988—such as path dependence, nonlinear feedback, and emergent macroeconomic behavior arising from micro-level interactions—are now central themes in complexity economics. At the same time, traditional equilibrium-based modeling remains deeply embedded in economic theory and practice, reflecting the continuing tension between analytical tractability and descriptive realism.

The purpose of revisiting this paper today is not to claim priority for ideas that have since been developed more rigorously by others. Rather, it is to illustrate how early exposure to nonlinear dynamics suggested a different way of thinking about economic systems—one in which stability does not necessarily imply equilibrium, and in which shocks may act as perturbations to trajectories rather than permanent structural shifts.

Viewed from the perspective of modern complexity economics, the original paper can be seen as an early attempt to apply nonlinear dynamical thinking to economic systems at a time when such approaches were largely absent from the discipline.

The central intuition remains the same: economic systems are composed of many interacting components whose collective behavior cannot always be understood by examining individual relationships in isolation. In such systems, the trajectory of the whole may remain stable even as the individual components of the system continuously adjust.